**Tutorial - Class Activity**

**26 September, 2018 (Solution)**

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**Solution**

a) We plug the continuously compounded interest rate, the forward price, the initial index level and the time to expiration in years into the valuation formula and solve for the dividend yield:

*F*0,*T* = *S*0 × *e*(*r* −*δ*)×*T*

⇔ = *e*(*r* −*δ*)×*T*

⇔ = (*r* − *δ*) × *T*

⇔ *δ* = *r* − 

⇒ *δ* = 0.05 −  = 0.05 − 0.035 = 0.015

b) With a dividend yield of only 0.005, you expect that the forward price would be:

*F*0,*T* = *S*0 × *e*(*r* − *δ*)×*T* = 1,100 × *e*(0.05−0.005)×0.75 = 1,100 × 1.0343 = 1,137.759

Therefore, if we think the dividend yield is 0.005, we consider the observed forward price of $1,129.257 to be too cheap. We will therefore buy the forward and create a synthetic short forward, capturing a certain amount of $8.502. We engage in a **reverse cash and carry arbitrage**:

|  |  |  |
| --- | --- | --- |
| Description | Today | In nine months |
| Long forward | 0 | *ST* − $1,129.257 |
| Sell short tailed position in | $1,100 × 0.99626 | −*ST* |
| index | = $1,095.88 |  |
| Lend $1,095.88 | −$1,095.88 | $1,137.759 |
| TOTAL | 0 | $8.502 |

c) With a dividend yield of 0.03, you expect that the forward price would be:

*F*0,*T* = *S*0 × *e*(*r* −*δ*)×*T* = 1,100 × *e*(0.05−0.03)×0.75 = 1,100 × 1.01511 = 1,116.62

Therefore, if we think the dividend yield is 0.03, we consider the observed forward price of $1,129.257 to be too expensive. We will therefore sell the forward and create a synthetic long forward, capturing a certain amount of $12.637. We engage in a **cash and carry arbitrage**:

|  |  |  |
| --- | --- | --- |
| Description | Today | In nine months |
| Short forward | 0 | $1,129.257 − *ST* |
| Buy tailed position in | −$1,100 × .97775 | *ST* |
| index | = −$1,075.526 |  |
| Borrow $1,075.526 | $1,075.526 | −$1,116.62 |
| TOTAL | 0 | $12.637 |

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**Solution**

a)

|  |  |  |
| --- | --- | --- |
| Description | Today | At expiration of the contract |
| Long forward | 0 | *ST* − *F*0*,T* = *ST* − *S*0*erT* |
| Lend *S*0 | −*S*0 | *S*0*erT* |
| Total | −*S*0 | *ST* |

In the first row, we made use of the forward price equation if the stock does not pay dividends. We see that the total aggregate position is equivalent to the payoff of one stock.

b) In the case of discrete dividends, we have:

|  |  |  |
| --- | --- | --- |
| Description | Today | At expiration of the contract |
| Long forward | 0 | *ST* − *F*0,*T* = *ST* − *S*0*erT* + |
| Lend *S*0 − | −*S*0 + | +*S*0*erT* − |
| Total | −*S*0 + | *ST* |

In the first row, we made use of the forward price equation if the stock pays discrete dividends. We see that the total aggregate position is equivalent to the payoff of one stock at time T.

c) In the case of a continuous dividend, we have to tail the position initially. We, therefore, create a synthetic share at the time of expiration T of the forward contract.

|  |  |  |
| --- | --- | --- |
| Description | Today | At expiration of the contract |
| Long forward | 0 | *ST* − *F*0,*T* = *ST* − *S*0*e*(*r* −*δ*)*T* |
| Lend *S*0*e*−*δT* | −*S*0*e*−*δT* | *S*0*e*(*r* −*δ*)*T* |
| Total | −*S*0*e*−*δT* | *ST* |

In the first row, we made use of the forward price equation if the stock pays a continuous dividend. We see that the total aggregate position is equivalent to the payoff of one stock at time T.